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LETTER TO THE EDITOR

$N = 4$ super-Liouville equation

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Abstract. We present a superfield $N = 4$ superextension of the Liouville equation with gauge $SU(2)\times SU(2)$ symmetry. It is formulated in terms of real quaternionic $N = 4$ superfield subjected to certain Grassmann analyticity constraints and possesses a zero-curvature representation on superalgebra su(1, 1|2). A possible relevance of the obtained system to the SU(2)-superstring is discussed.

Supersymmetric extensions of the two-dimensional Liouville equation

$$u_{+-} = m^2\, e^{-2u} \qquad (u_{+-} \equiv \partial_+\partial_- u, [m] = L^{-1}) \tag{1}$$

are interesting mainly in view of their possible relation to superstrings and four-dimensional supergauge theories (Ademollo *et al* 1976a, b, Brink and Schwarz 1977, Polyakov 1981).

The simplest $N = 1$ extension of equation (1) was treated from different standpoints (Polyakov 1981, Chaichian and Kulish 1978, Leznov *et al* 1980). The superfield (SF) theory of the $N = 2$ super-Liouville equation (Lagrangian, general solution, zero-curvature representation) has been worked out in our paper (Ivanov and Krivonos 1983). It is described by a complex analytic $N = 2$ SF and possesses internal gauge $U_+(1)\times U_-(1)$ symmetry.

Here we present the next $N = 4$ superextension of equation (1) with the nonabelian gauge group $SU(2)_+ \times SU_-(2)$. This extension is unique in that the relevant supermultiplet (4+4 components on-shell) is the maximally possible one containing the dilation $u(x)$ and including no fields with anomalous conformal dimensions. The basic object is a real quaternionic $N = 4$ SF subjected to irreducibility constraints of the hypermultiplet type (Fayet 1976, Sohnius *et al* 1981, Sohnius 1978). We analyse the invariance properties of the system obtained and construct for it a zero-curvature representation in $N = 4$ superspace (SS). It is demonstrated that the $N = 4$ super-Liouville equation exhibits invariance (at the classical level) with respect to the transformation of an infinite dimensional SU(2) string superalgebra (SA) of Ademollo *et al* (1976c).

We exploit the same general group-theoretic approach as in the $N = 2$ case (Ivanov and Krivonos 1983). It deals with nonlinear realisations of infinite-dimensional (super)symmetries and has been explained in our previous papers (Ivanov and Krivonos 1983, 1984 a, b).

The key points of our method consist of choosing an infinite dimensional SA $\mathscr{G}$, for which the nonlinear realisation is constructed, and secondly its subalgebras: the vacuum stability subalgebra $\mathscr{H}$ and zero-curvature representation subalgebra $\mathscr{G}_0$. In the present case, $\mathscr{G}$ is taken to be a direct sum of two contact SA's $\mathbb{K}^A_\pm(1|2)$ with the following structure relations (Ademollo *et al* 1976c)†:

$$
\begin{aligned}
&\mathrm{i}[L_\pm^n, L_\pm^m] = (n-m)L_\pm^{n+m},\\
&\{G_{\alpha\pm}^r, \bar{G}_\pm^{s\beta}\} = -2\delta_\alpha^\beta L_\pm^{r+s} + 2(r-s)(\sigma^k)_\alpha^\beta T_{k\pm}^{r+s},\\
&\mathrm{i}[L_\pm^n, T_{k\pm}^p] = -pT_{k\pm}^{p+n},\\
&\mathrm{i}[L_\pm^n, G_{\alpha\pm}^r] = (\tfrac{1}{2}n - r)G_{\alpha\pm}^{r+n},\\
&\mathrm{i}[T_{k\pm}^p, G_{\alpha\pm}^r] = -\tfrac{1}{2}(\sigma_k)_\alpha^\beta G_{\beta\pm}^{r+p},\\
&[T_{k\pm}^p, T_{i\pm}^l] = \varepsilon_{kij}T_{j\pm}^{p+l},\\
&\{G_{\alpha\pm}^r, G_{\beta\pm}^s\} = 0,\\
&\{\bar{G}_\pm^{r\alpha}, \bar{G}_\pm^{s\beta}\} = 0,
\end{aligned}
$$

($n, m = -1, 0, 1, \ldots; r, s = -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \ldots; p, l = 0, 1, 2, \ldots; \alpha, \beta = 1, 2; i, j, k = 1, 2, 3$). The even sector of the SA(2) includes, besides the conformal algebra of *the two-dimensional world* $\mathbb{K}_\pm(1) = \{L_\pm^n\}$ two gauge algebras $\{T_{k\pm}^p\}$ with the local part $\mathrm{su}_1(2)\oplus\mathrm{su}_2(2) = \{T_{k\pm}^0\}$. The algebra $\mathrm{su}_+(2)\oplus\mathrm{su}_-(2)$ is the automorphism algebra of the rigid $N=4$ SA with generators $L_\pm^{-1}$ (translations), $U = L_+^0 - L_-^0$ (so(1, 1)-pseudorotations) and $G_{\alpha\pm}^{-1/2}$, $\bar{G}_\pm^{-1/2\alpha}$ (supertranslations).

Further, one constructs by $\mathscr{G}$ the supergroup G and considers a nonlinear realisation of G in its certain coset space G/H. The coset with a minimal number of essential parameters (i.e. those through which all others can be covariantly expressed) corresponds to the choice of $\mathrm{H} = \mathrm{SO}(1, 1)\times\mathrm{SU}(2)$ with generators U, $T_k^0 = T_{k+}^0 + T_{k-}^0$. The essential parameters are the $N=4$ SS coordinates $x^\pm$, $\theta^{\alpha\pm}$, $\bar{\theta}_\alpha^\pm$ and SF's $u(x, \theta, \bar{\theta})$, $\varphi^k(x, \theta, \bar{\theta})$ associated, respectively, with generators $L_\pm^{-1}$, $G_{\alpha\pm}^{-1/2}$, $\bar{G}_\pm^{-\alpha/2}$ and $L_+^0 + L_-^0$, $T_{k+}^0 - T_{k-}^0$.

The next steps are to define the Cartan one-form Ω on the SA $\mathscr{G}$ and to perform a covariant reduction of Ω to a one-form Ω_0^{Red} which obeys the zero-curvature condition on certain SA $\mathscr{G}_0 \subset \mathscr{G}$:

$$\Omega = \Omega_0^{\mathrm{Red}} \in \mathscr{G}_0 \tag{3}$$

$$\mathrm{d}\Omega_0^{\mathrm{Red}} - \Omega_0^{\mathrm{Red}} \wedge \Omega_0^{\mathrm{Red}} = 0. \tag{4}$$

The zero-curvature representation (4) follows from the reduction constraint (3) and the standard Maurer–Cartan equation for the initial one-form Ω. We choose as $\mathscr{G}_0$ the $N=4$ superextension su(1, 1|2) of the algebra sl(2, R), with the following generators

$$
\begin{aligned}
&R_\pm = L_\pm^{-1} + m^2 L_\mp^1, && Q_{\alpha\pm} = G_{\alpha\pm}^{-1/2} \pm mG_{\alpha\mp}^{1/2},\\
&U = L_+^0 - L_-^0, && \bar{Q}_\pm^\alpha = \bar{G}_\pm^{-\alpha/2} \pm m\bar{G}_\mp^{\alpha/2},\\
&T_i^0 = T_{i+}^0 + T_{i-}^0,
\end{aligned}
\tag{5}
$$

where U, R_+, R_- form sl(2, R). In what follows we need to know explicitly only the spinor components of Ω_0^{Red}.

The covariant reduction constraint (3) produces an infinite set of the Pfaff's equations for the components of Ω_0^{Red}. They express the SF parameters of G/H in

† The whole SU(2) string SA (Ademollo *et al* 1976c) contains in addition the generators with any negative conformal dimensions and also c-number central charges of the pure quantum origin.

terms of $u(x, \theta, \bar{\theta})$ and $\varphi^i(x, \theta, \bar{\theta})$ and result, owing to (4), in the dynamical constraints on u, φ^i which are just the desirable $N=4$ extension of the Liouville equation:

$$\mathscr{D}_-^{(\alpha} q_\gamma^{\beta)} = 0, \qquad \bar{\mathscr{D}}_{+(\alpha} q_{\gamma)}^{\beta} = 0 \tag{6}$$

$$\begin{aligned} &\mathscr{D}_-^\alpha (q \mathscr{D}_+^\gamma q^{-1})_\gamma^\beta = 0 \\ &\mathscr{D}_+^\alpha (q^{-1} \bar{\mathscr{D}}_{-\gamma} q)_\beta^\gamma + 4\mathrm{i} m \bar{q}_\beta^\alpha = 0. \end{aligned} \tag{7}$$

Here, u and φ^i are combined into the single real quaternionic SF:

$$\begin{aligned} &q_\alpha^\beta \equiv (\mathrm{e}^{-u-\mathrm{i}\varphi\cdot\sigma})_\alpha^\beta, \\ &\bar{q}_\alpha^\beta \equiv (\mathrm{e}^{-u+\mathrm{i}\varphi\cdot\sigma})_\alpha^\beta = -\varepsilon_{\alpha\gamma}\varepsilon^{\beta\delta} q_\delta^\gamma, \end{aligned} \tag{8}$$

and $N=4$ covariant spinor derivatives are introduced:

$$\begin{aligned} &\mathscr{D}_\pm^\alpha = \mathrm{i}\theta^{\alpha\pm}\partial/\partial x^\pm + \partial/\partial\bar{\theta}_\alpha^\pm, \qquad \bar{\mathscr{D}}_{\pm\alpha} = \mathrm{i}\bar{\theta}_\alpha^\pm \partial/\partial x^\pm + \partial/\partial\theta^{\alpha\pm}, \\ &\{\mathscr{D}_\pm^\alpha, \bar{\mathscr{D}}_{\pm\beta}\} = 2\mathrm{i}\delta_\beta^\alpha \partial/\partial x^\pm. \end{aligned} \tag{9}$$

Relations (6) are the irredicibility conditions for q_α^β. They directly generalise the Grassmann analyticity conditions of the $N=2$ case (Ivanov and Krivonos 1983). One easily checks their consistency with the dynamical equations (7). Because of the reality of q_α^β, these relations hold also with the conjugated spinor derivatives. Such constraints are well known in $D=4$; they define there the simplest representation of $N=2$ SUSY, the hypermultiplet (Fayet 1976, Sohnius 1978, Sohnius *et al* 1981). This correspondence can be understood from the fact that $N=4$ SUSY in $D=2$ and $N=2$ SUSY in $D=4$ are related by dimensional reduction.

The irreducible content of q_β^α is given by eight bosons

$$\begin{aligned} &(q_0)_\alpha^\beta = q_\alpha^\beta|_{\theta=\bar{\theta}=0}, \\ &C_1 = \mathscr{D}_{-\alpha}\mathscr{D}_+^\beta q_\beta^\alpha|_{\theta=\bar{\theta}=0}, \qquad C_2 = \bar{\mathscr{D}}_{-\alpha}\mathscr{D}_+^\beta q_\beta^\alpha|_{\theta=\bar{\theta}=0} \end{aligned}$$

and eight fermions

$$\begin{aligned} &\psi_-^\alpha = -(\bar{q}\mathscr{D}^\beta \bar{q}^{-1})_\beta^\alpha|_{\theta=\bar{\theta}=0}, \\ &\chi_+^\alpha = (\mathscr{D}_+^\beta q q^{-1})_\beta^\alpha|_{\theta=\bar{\theta}=0}. \end{aligned}$$

The fields C_1, C_2 are auxiliary, therefore q_α^β contains on-shell $4+4$ components that exactly coincide with the content of the hypermultiplet.

The component equations are obtained by a successive action of spinor derivatives on (7) with a subsequent extraction of $\theta, \bar{\theta}$-independent terms in the resulting expression. In this way, we find

$$\begin{aligned} &C_1 = -(\chi_+ q_0 \psi_-), \\ &C_2 = (\chi_+ q_0 \bar{\psi}_-) - \tfrac{1}{4}\mathrm{i}m\, \mathrm{Sp}(\bar{q}_0 q_0), \\ &\partial_+ \psi_-^\alpha = -m(\chi_+ q_0)^\alpha, \\ &\partial_- \chi_+^\alpha = m(\psi_- \bar{q}_0)^\alpha, \\ &\partial_+(q_0^{-1}\partial_- q_0)_\alpha^\beta = -m^2(\bar{q}_0 q_0)_\alpha^\beta - \tfrac{1}{8}\mathrm{i}m[\delta_\alpha^\beta(\psi_- \bar{q}_0 \chi_+) - (\bar{q}_0\chi_+)_\alpha \bar{\psi}_-^\beta - \psi_{-\alpha}(\bar{\chi}_+ q_0)^\beta]. \end{aligned} \tag{10}$$

The bosonic sector of the system (10) upon eliminating fermionic fields is described by the equation

$$\partial_+(q_0^{-1}\partial - q_0)_\alpha^\beta = -m^2(\bar{q}_0 q_0)_\alpha^\beta \tag{11}$$

that divides into two independent equations, one of which is the ordinary Liouville equation for $u(x) = u(x, \theta, \bar{\theta})|_{\theta=\bar{\theta}=0} = -\frac{1}{4}\,\mathrm{Sp}\ln(\bar{q}q)$ (it corresponds to the trace of (11)), and the other is

$$\partial_+ \mathrm{Sp}(q\sigma^i\partial_- q^{-1}) = 0. \tag{12}$$

In contrast to the standard equation of nonlinear σ-model for the principal field on the group SU(2), equation (12) has no term with interchanged ∂_+ and ∂_-. This difference entails some radical consequences. First, equation (12) possesses gauge $\mathrm{SU}_+(2)\otimes \mathrm{SU}_-(2)$ symmetry; it is invariant under transformations

$$g(x) \to \mathrm{e}^{\mathrm{i}a(x^+)}\, g(x)\, \mathrm{e}^{\mathrm{i}b(x^-)} \tag{13}$$

and, as a result, can be explicitly solved:

$$g(x) = g_{\mathrm{I}}(x^+)g_{\mathrm{II}}(x^-). \tag{14}$$

Secondly, equation (12) and respectively the full system (10) are of a non-Lagrangian type (at least, in the variables through which they are written here). A more detailed discussion of these properties of equation (12) and a method of transforming it to a Lagrangian form can be found in our forthcoming paper.

Equations (6) and (7) possess, by construction, an infinite-parameter symmetry with respect to the SU(2) superconformal supergroup G constructed by SA $\mathscr{G}$ and realised via left shifts in the coset space G/H. A direct calculation yields a realisation of G on the $N=4$ SS coordinates and SF q_α^β:

$$\partial x^\pm = \mathrm{e}^{\mathrm{i}(\theta^\pm\bar{\theta}^\pm)\partial_\pm}[\tfrac{1}{2}f^\pm + \mathrm{i}\theta^\pm\bar{\zeta}^\pm] + \mathrm{e}^{-\mathrm{i}(\theta^\pm\bar{\theta}^\pm)\partial_\pm}[\tfrac{1}{2}f^\pm - \mathrm{i}\zeta^\pm\bar{\theta}^\pm], \tag{15}$$

$$\delta\theta^{\alpha\pm} = \mathrm{e}^{\mathrm{i}(\theta^\pm\bar{\theta}^\pm)\partial_\pm}[\zeta^{\alpha\pm} + \tfrac{1}{2}f^{\pm\prime}\theta^{\alpha\pm} + 2\mathrm{i}(\theta^\pm\bar{\zeta}^{\pm\prime})\theta^{\alpha\pm} + \tfrac{1}{2}\mathrm{i}a^{k\pm}(\theta^\pm\sigma^k)^a], \tag{16}$$

$$\begin{aligned} \partial q_\alpha^\beta &= W_\alpha^{+\rho} q_\rho^\beta + q_\alpha^\gamma W_\gamma^{-\beta}, \\ W_\alpha^{+\rho} &= \tfrac{1}{2}(\mathscr{D}^{+\rho}\delta\bar{\theta}_\alpha^+ - \bar{\mathscr{D}}_\alpha^+\delta\theta^{+\rho} - \delta_\alpha^\rho\,\mathscr{D}^{+\lambda}\delta\bar{\theta}_\lambda^+) \\ W_\gamma^{-\beta} &= \tfrac{1}{2}(\bar{\mathscr{D}}_\gamma^-\delta\theta^{-\beta} - \mathscr{D}^{-\beta}\delta\bar{\theta}_\gamma^- - \delta_\alpha^\beta\bar{\mathscr{D}}_\lambda^-\delta\theta^{-\lambda}) \end{aligned} \tag{17}$$

$f^\pm(x^\pm)$, $\zeta^{\alpha\pm}(x^\pm)$, $a^{k\pm}(x^\pm)$ being, respectively, infinitesimal parameters of conformal, local supersymmetric, and local $\mathrm{SU}_\pm(2)$ transformations. The coordinate transformations (15), when written in terms of complex variables $\xi^\pm \equiv x^\pm + \mathrm{i}(\theta^\pm\bar{\theta}^\pm)$ or $(\xi^\pm)^+$, cooincide with those given in (Ademollo *et al* 1976b). It is a simple task to check that under these transformations the covariant differentials:

$$\mathrm{i}\Delta x^\pm = \mathrm{d}x^\pm + \mathrm{i}(\theta^{\pm\alpha}\,\mathrm{d}\bar{\theta}_\alpha^\pm - \mathrm{d}\theta^{\pm\alpha}\bar{\theta}_\alpha^\pm)$$

are multiplied by some superfunctions, in accord with the definition of contact SA's (Kac 1977).

As has been given above, system (6), (7) is equivalent to requiring the curvature of the one-form Ω_0^{Red} to vanish. It means that this system can be interpreted as the integrability condition for some linear problem in $N=4$ SS. The minimal linear set can be constructed starting only with the spinor components of Ω_0^{Red}:

$$
\begin{aligned}
\Omega_+^\alpha &= -\tfrac{1}{4}\mathrm{i}(q\mathscr{D}_+^\beta q^{-1})_\beta^\alpha U + \mathrm{i}(\tilde{q}\mathscr{D}_+^\alpha \tilde{q}^{-1}\sigma^k) T_0^k + \overline{(\tilde{q})}{}_\beta^\alpha \bar{Q}^{+\beta}, \\
\bar{\Omega}_{+\alpha} &= \tfrac{1}{4}\mathrm{i}(\bar{q}^{-1}\bar{\mathscr{D}}_{+\beta}\bar{q})_\alpha^\beta U - \mathrm{i}(\tilde{q}\bar{\mathscr{D}}_{+\alpha}\tilde{q}^{-1}\sigma^k) T_0^k + (\tilde{q})_\alpha^\beta Q_\beta^+, \\
\bar{\Omega}_{-\alpha} &= -\tfrac{1}{4}\mathrm{i}(q^{-1}\bar{\mathscr{D}}_{-\beta} q)_\alpha^\beta U + \mathrm{i}(\tilde{q}^{-1}\bar{\mathscr{D}}_{-\alpha}\tilde{q}\sigma^k) T_0^k + \overline{(\tilde{q})}{}_\alpha^\beta Q_\beta^-, \\
\Omega_-^\alpha &= \tfrac{1}{4}\mathrm{i}(\bar{q}\mathscr{D}_-^\beta \bar{q}^{-1}) U - \mathrm{i}(\tilde{q}^{-1}\mathscr{D}^{-\alpha}\tilde{q}\sigma^k) T_0^k + (\tilde{q})_\beta^\alpha \bar{Q}^{-\beta}
\end{aligned}
\tag{18}
$$

where

$$\tilde{q}_\alpha^\beta = (\exp(-\tfrac{1}{2}(u + \mathrm{i}\varphi\cdot\sigma)_\alpha^\beta.$$

Let us define 'lengthened' spinor derivatives

$$\nabla_\pm^\alpha = \mathscr{D}_\pm^\alpha + \Omega_\pm^\alpha, \qquad \bar{\nabla}_{\pm\alpha} = \bar{\mathscr{D}}_{\pm\alpha} + \bar{\Omega}_{\pm\alpha}. \tag{19}$$

One may easily see that system (6), (7) is equivalent to the constraints:

$$\{\nabla_-^\alpha, \nabla_-^\beta\} = \{\nabla_-^\alpha, \bar{\nabla}_{+\beta}\} = \{\nabla_-^\alpha, \nabla_+^\beta\} = 0. \tag{20}$$

All other commutators and anticommutators (except for $\{\nabla_\pm^\alpha, \bar{\nabla}_{\pm\beta}\}$ which are in fact the definition of the 'lengthened' vector derivatives) vanish as a consequence of (20). Notice that it is not necessary to know the full set of the structure relations of su(1, 1|2) when evaluating (20). The operators $\nabla_-^\alpha, \bar{\nabla}_{+\beta}$ are given on its graded subalgebra $\{U, T_0^k, Q_{\alpha+}, \bar{Q}_-^\beta\}$ and $\nabla_+^\beta, \bar{\nabla}_{-\alpha}$ on the conjugated one $\{U, T_0^k, \bar{Q}_+^\alpha, Q_{\beta-}\}$. Though these subalgebras do not commute and close on the whole su(1, 1|2) the only crossing anticommutator between them which enters into equations (20) is the following

$$\{Q_{+\alpha}, Q_{-\beta}\} = 0.$$

Clearly, it does not lead one out of the above subalgebras.

The simplest linear problem can be written for the fundamental representation of su(1, 1|2) and thus has the matrix dimension 4×4:

$$\nabla_\pm^\alpha V = \bar{\nabla}_{\pm\beta} V = 0 \tag{21}$$

with V being a row of four complex $N=4$ SF's. Spectral parameters can be introduced into (21) as in the $N=0$, $N=1$, and $N=2$ cases (Ivanov and Krivonos 1983, 1984a, b), by a constant right H-transformation of the coset G/H. Since $\mathrm{H} = \mathrm{SO}(1,1)\times\mathrm{SU}(2)$ in the present case, spectral parameters form now a four-dimensional real manifold (they can be combined into a real quaternion $\lambda_0 + \mathrm{i}\lambda^k\sigma^k$).

A few words concerning the geometric interpretation of equations (6), (7) are to the point. Representation (4) can be treated as the dynamically emerging Maurer–Cartan equatin for the homogeneous SS SU(1, 1|2)/SO(1, 1)×SU(2). This curved SS is the $N=4$ superextension of the two-dimensional pseudosphere SO(1, 2)/SO(1, 1). Any classic solution of the $N=0$ Liouville equation (1) provides a particular parametrisation of this pseudosphere. Analogously, any solution of (6), (7) specifies a choice of parameters on the pseudosupersphere SU(1, 1|2)/SO(1, 1)×SU(2).

The equations we have constructed yield a realisation of the SU(2) superstring SA different from the standard realisation considered by Ademollo *et al* (1976b). Though the number of physical components of the $N=4$ Liouville supermultiplet coincides with that of the basic SU(2) superstring multiplet in the formulation of Ademollo *et al*, these components have essentially different properties with respect to the diagonal automorphism group SU(2). Indeed, the physical bosons in the case of SU(2) superstring are SU(2) singlets while in our scheme they form a reducible $1 \oplus 3$ SU(2) multiplet. Keeping in mind this difference, it seems interesting to quantise system (10) and to learn which version of the SU(2) superstring will appear therewith. It may happen, e.g., that such a theory is free of the main difficulty of the standard formulation, the presence of ghosts in any space–time dimension. To carry out this programme, it would be desirable to convert system (10) into Lagrangian form.

In conclusion, let us stress that the $N=1$, $N=2$, and $N=4$ super-Liouville equations are formulated most naturally in terms of real, complex, and quaternionic SF's subjected in the last two cases to proper Grassman analyticity constraints. This correspondence can be looked upon as one more argument in favour of a profound intrinsic connection between supersymmetries and systems of hypercomplex numbers.

We cordially thank D A Leites for an enlightening discussion of the structure of contact superalgebras.

Note added. L D Faddeev kindly drew our attention to the fact that equations of type (15) have some (singular) Lagrangian even in the original variables. It can be obtained by adding, to the standard σ-model Lagrangian, the so-called Wess–Zumino term (Wess and Zumino 1974, Novikov 1981). σ-models of this type are intensively discussed for the last time in literature. For instance, they naturally appear in the problem of non-abelian bosonisation (Witten 1984). We now expect that the whole system (13) also possesses a Lagrangian which can be constructed via a proper supersymmetrisation of the Wess–Zumino term. A more detailed treatment of these problems will be given elsewhere.

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